# Asynchronous Execution and Communication Latency in Distributed Constraint Optimization

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Outline:

- Motivation: real-time coordination of sensors in a high-latency network
- Modeling coordination as graph colouring
- Soft graph colouring for real-time responsiveness
- A class of distributed anytime algorithms (synchronous)
- Convergence
- Tightness of constraints: conservative variant
- Scalability and robustness
- Asynchronous execution
- Very high communication latencies

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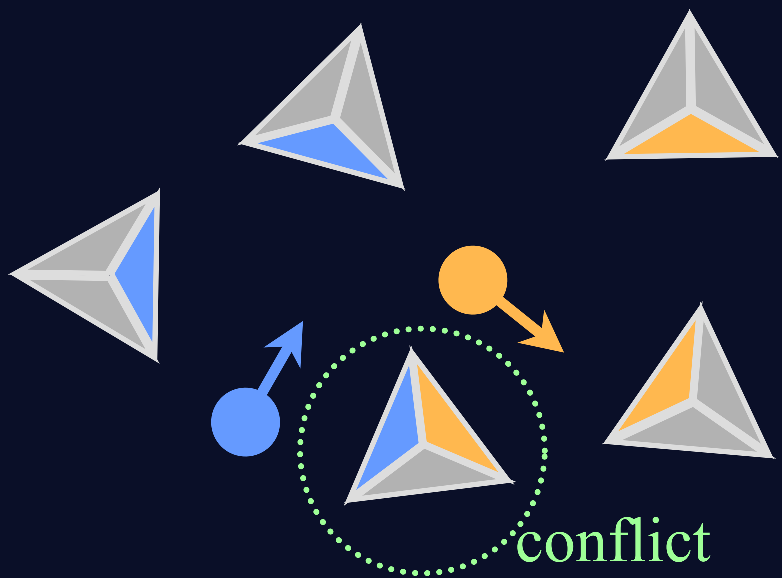
# Motivation: Large Networks of Short-Range Sensors

- Short-range, directional radars
  - each can scan 1 of its 3 sectors at a time
  - each scan acquires range & radial velocity
  - battery-operated – conservation important
- Collaboration needed for tracking
  - 3 approximately-simultaneous scans needed for trilateralization
- Low-power radio communication
  - low bandwidth, high latency
  - reveals positions of radars – minimize

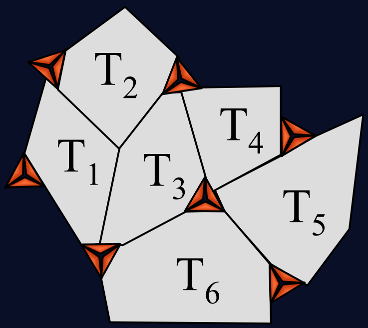



- Coordination mechanism organizes collaboration
  - optimizes simultaneous scanning, minimizes costs
- Must be:
  - scalable (e.g., to $10^5$ sensors)
  - real-time adaptive (e.g., new targets are detected, existing targets disappear)
  - robust (e.g., hardware may fail)

# Inter-Sensor Collaboration

- Main requirement: scan each target simultaneously with 3 radars
  - define virtual resources: *trackers*
  - each tracker is comprised of 3 sectors on nearby radars
    - $T_i \equiv \{R_{i1}:S_{i1}, R_{i2}:S_{i2}, R_{i3}:S_{i3}\}$
  - each tracker can track a single target over some contiguous region
- Main constraint: each radar can scan only 1 sector at a time
  - if two trackers use different sectors on the same radar, they are mutually exclusive
    - $\text{mutually\_exclusive}(T_1, T_2) \Leftrightarrow \exists\, j,k \in \{1, 2, 3\}: R_{1j}=R_{2k} \wedge S_{1j} \neq S_{2k}$
- Compute a cyclic schedule of tracker usage
  - worst-case assumption: all trackers need to be used
  - mutually exclusive trackers cannot be used in the same time slot
  - number of time slots determined by target speed, scan time & revisit period

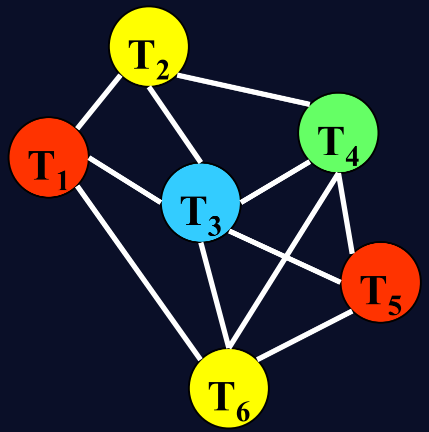
| timeslot # | scan start time (seconds) | scan end time (seconds) | T1 | T2 | T3 | T4 | T5 | T6 |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.0 | 2.0 | X | | | | X | |
| 2 | 2.0 | 4.0 | | X | | | | X |
| 3 | 4.0 | 6.0 | | | X | | | |
| 4 | 6.0 | 8.0 | | | | X | | |



# Modeling Coordination as Graph Colouring

- Each tracker can be mapped to a *node* in an undirected graph
- Each mutual exclusion constraint then maps to an *edge*
  - nodes that are *adjacent* in the graph are mutually exclusive/cannot be used simultaneously
  - two nodes are said to be neighbors iff they are adjacent
- A *proper k-colouring* of the graph's nodes maps to a feasible schedule
  - time slot ⇔ integer in $Z_k$ ⇔ colour

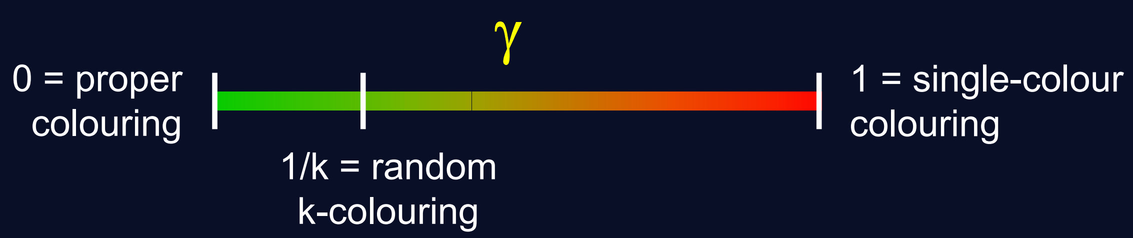
| timeslot # | scan start time (seconds) | scan end time (seconds) | T1 | T2 | T3 | T4 | T5 | T6 |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.0 | 2.0 | | | | | | |
| 2 | 2.0 | 4.0 | | | | | | |
| 3 | 4.0 | 6.0 | | | | | | |
| 4 | 6.0 | 8.0 | | | | | | |



# Soft Graph Colouring

- An edge connecting nodes of the same colour represents a *conflict*
  - some radar has been scheduled to scan two sectors simultaneously
- For real-time adaptation, the number of conflicts must be quickly reduced
  - fast reduction to acceptable levels is more important than total elimination
- Define the *degree of conflict* as the fraction of edges that are conflicts
  - let E be the set of edges and $C_v$ the colour of node v

$$\gamma \equiv \frac{\left|\left\{\{u,v\} \in E \mid C_u = C_v\right\}\right|}{|E|}$$



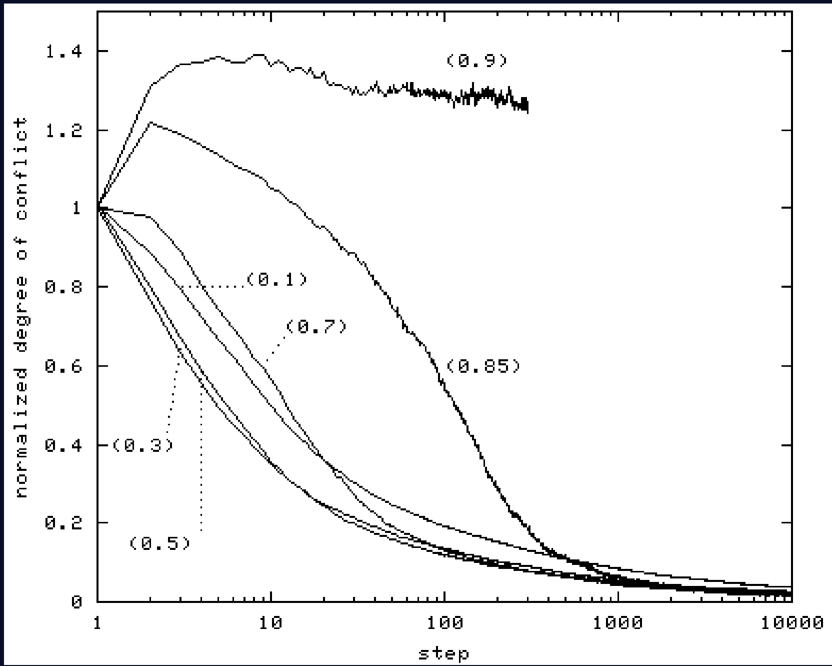

- Normalize: $\Gamma \equiv k\gamma$
  - random k-colouring has an expected $\Gamma$ of 1
- Assessment of coordination mechanism is based on how quickly it reduces $\Gamma$ after random initialization

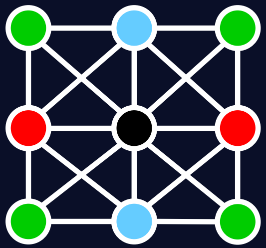
# A Class of Distributed Anytime Algorithms (synchronous)

- Main idea: each node repeatedly chooses its own colour to minimize its conflicts with neighbouring nodes
- Fixed Probability algorithm FP(p) …
  - Initialization:
    - each node chooses a random colour and informs its neighbours
  - Synchronized infinite loop:
    - probabilistic activation
      - a node activates if a randomly generated number falls below some fixed activation level p
    - if a node activates, it non-deterministically chooses its next colour
      - it computes a histogram of colour usage among its neighbours, based on what they last told it
      - it then chooses any colour that is least used in the histogram
      - if the chosen colour differs from its current colour, it tells its neighbours
- Convergence?
  - under the right conditions, the total number of conflicts reduces over time and *may* converge to 0 …

# Effect of Activation Level on Convergence of FP

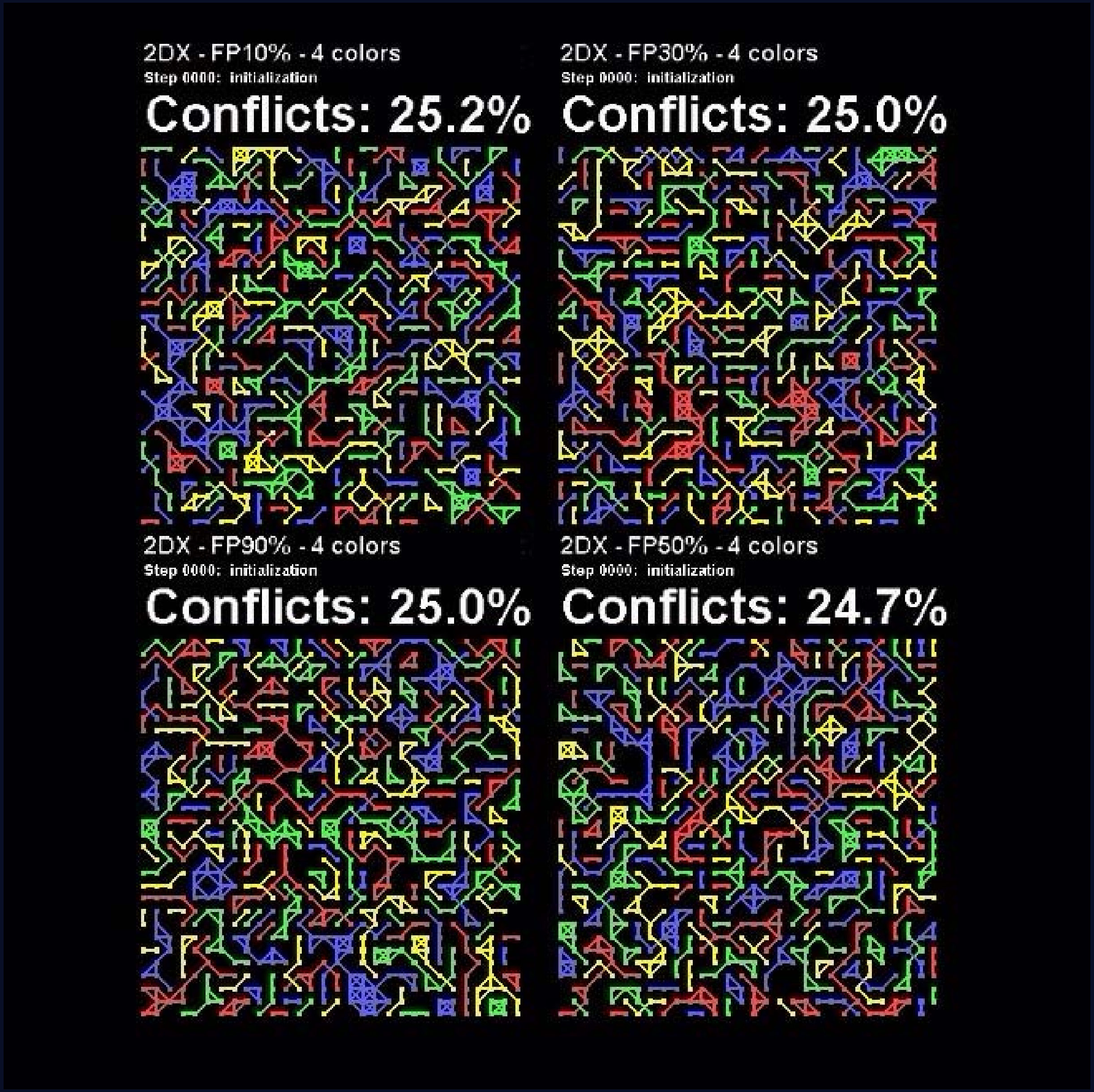
- Measure (normalized) degree of conflict after each synchronous step
  - experiment performed in simulator
- When activation level is too high, thrashing occurs
  - too many neighbours are simultaneously updating colours
  - because of out-of-date information, they make mutually harmful decisions
- When activation level is too low, adaptivity is hindered
  - extreme case is sequential execution
- Need compromise between speed and coherence
  - an activation level of 0.3 seems to be reasonable for sparse graphs
  - this level was used for experiments reported in following slides



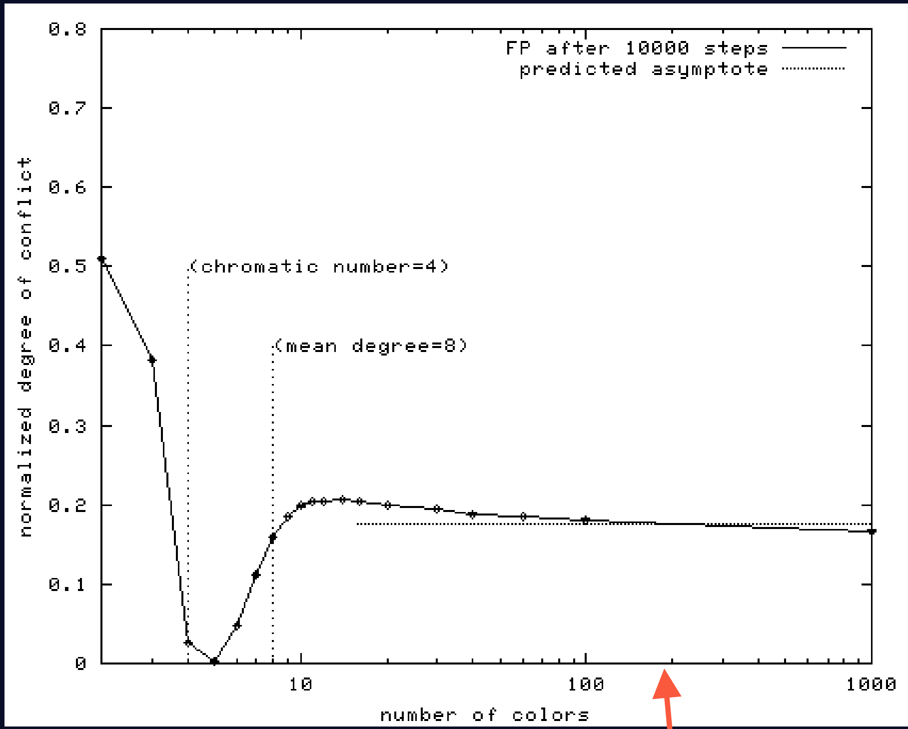
- experimental results shown for 2D grids
  - number of colours = chromatic number = 4
  - 500-5000 nodes
- experiments also performed with random graphs having higher, known chromatic numbers

# Animation: Activation Threshold



# Effect of Tightness of Constraints

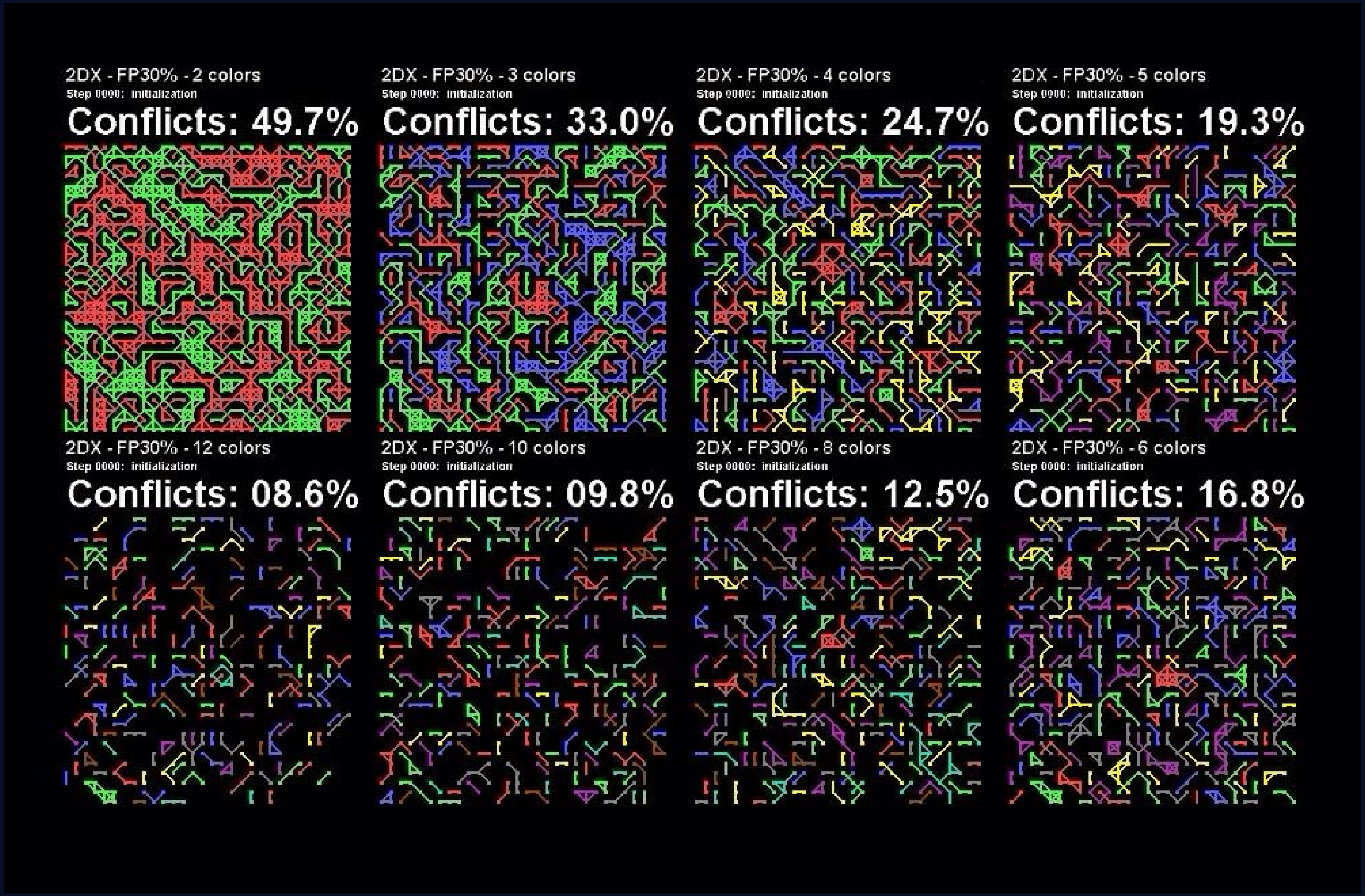
- Performance of FP is good on over-constrained problems
  - where #colours<chromatic number
  - for 2D & 3D grids, observed convergence value of degree of conflict is close to theoretical minimum
- Performance of FP is poor on loosely constrained problems
  - where #colours>>chromatic number
  - intuitively, these are easy problems
- When loosely constrained, each colour choice is essentially random
  - for each given node, most colours are not used by any neighbour
  - FP chooses randomly from among the unused colours
  - asymptotic value predicted as $\alpha/(2-\alpha)$ where $\alpha$ is the activation level




this is *not* a time axis

- experimental results shown for 2D grids
- chromatic number = 4

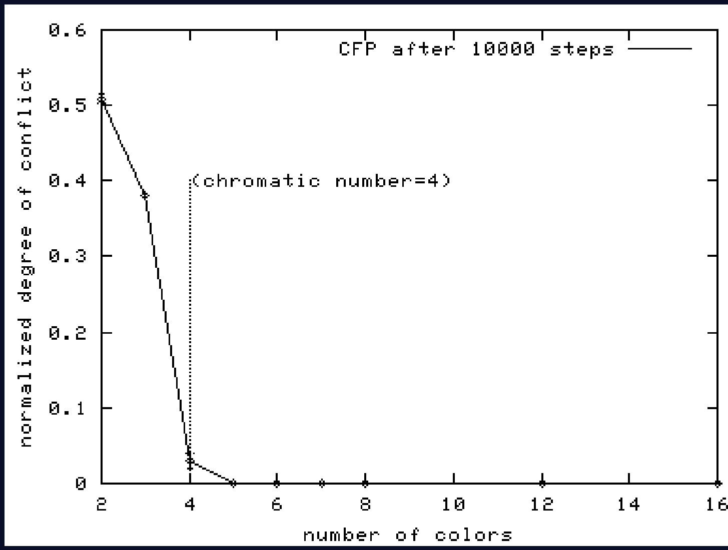
# Animation: Tightness of Constraints



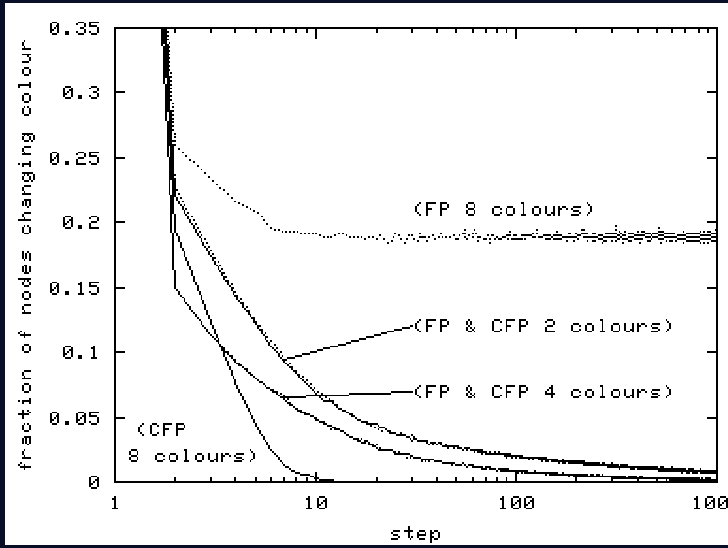
# CFP: Conservative Variant

- Colour choice is non-deterministic
- But activation is restricted
  – in addition to passing the test for random number<activation level, a node may activate *only* if it has a conflict with any neighbour
- Conservative variant has good performance overall
  – communication costs are also better than FP's for loosely constrained problems
    - under FP, node activity continues unabated forever
    - under CFP, node activity decreases with the degree of conflict

conflicts

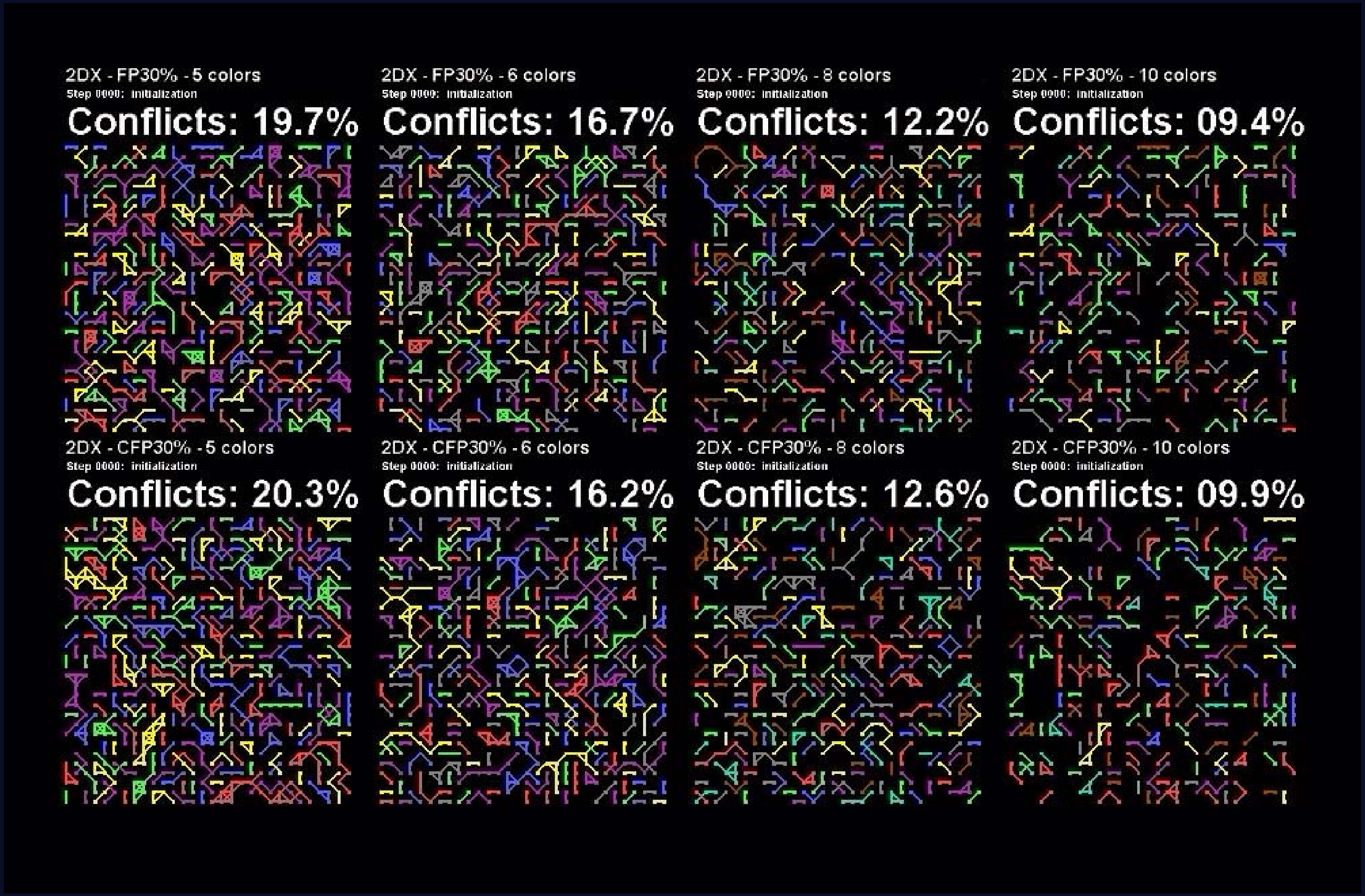



communication rate



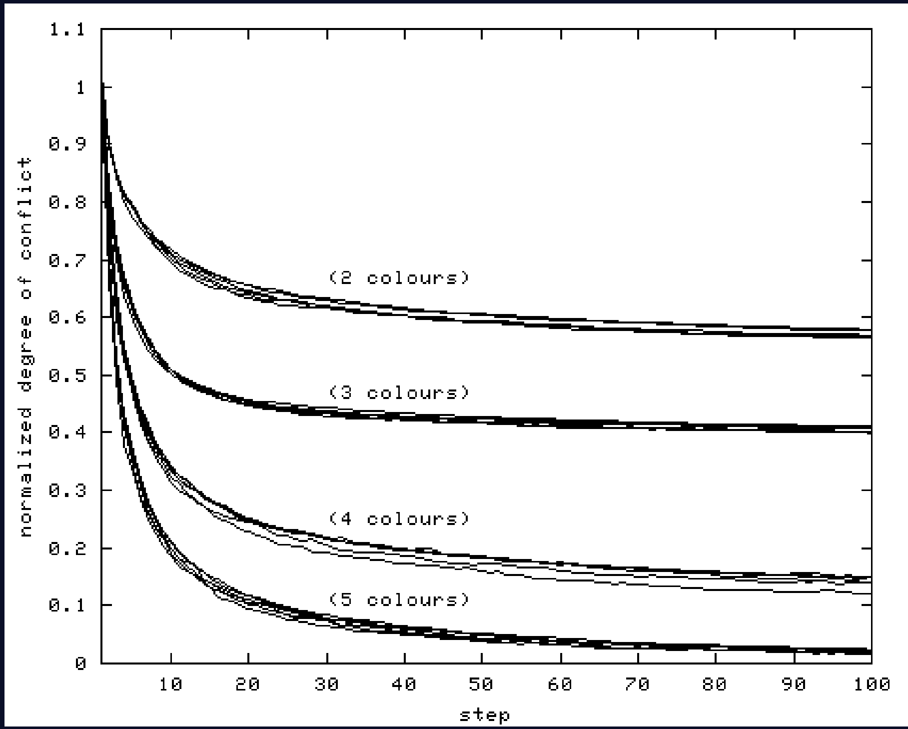

- experimental results shown for 2D grids
- chromatic number = 4

# Animation: FP vs. CFP



# Scalability

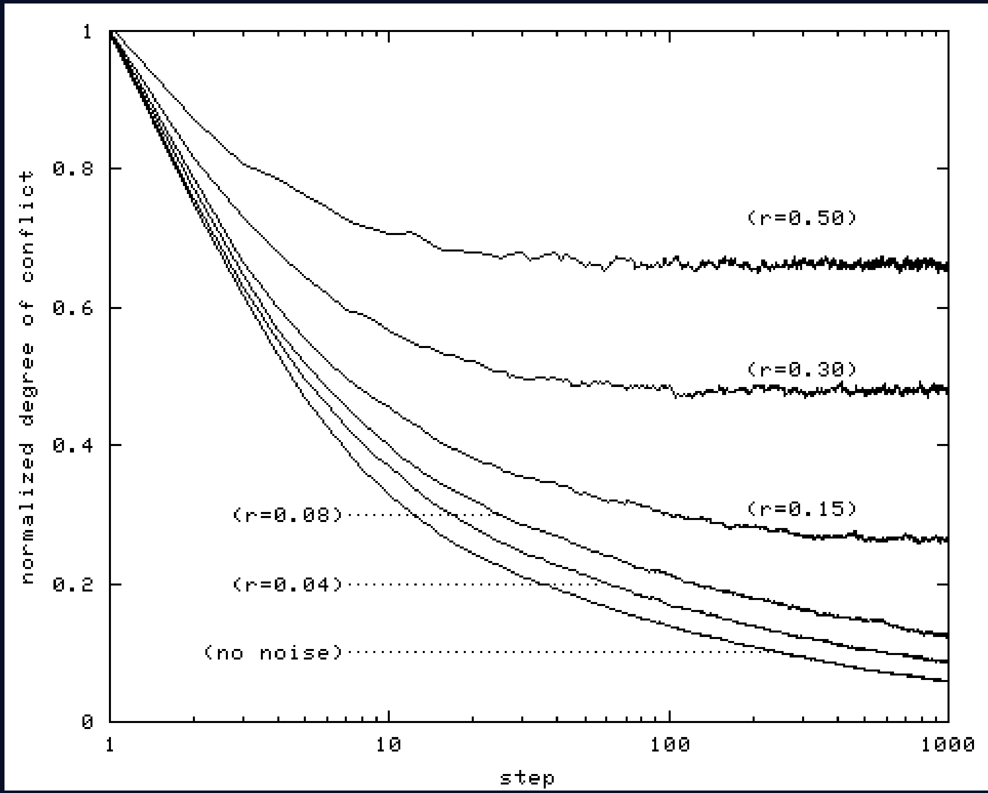
- The algorithm is scalable in cost
  - per node, per step costs depend on (mean) degree of the graph
  - they do not depend on the number of nodes
    - to the extent that the mean degree is independent of the number of nodes
- The algorithm is scalable in performance
  - for large graphs, the reduction in normalized degree of conflict over steps shows little variation for graphs of different sizes




- results shown are for CFP(0.3)
- 6 graphs of different sizes (500-5000 nodes)
  - each graph has chromatic number 4
  - each was coloured using 2, 3, 4 & 5 colours

# Robust against Communication Noise

- Each colour-change message subjected to random process:
  - probability r, colour randomized
  - probability d, message lost
  - otherwise, message unchanged
- For small amounts of noise, incremental increases in degree of conflict are observed
  - no catastrophic failure



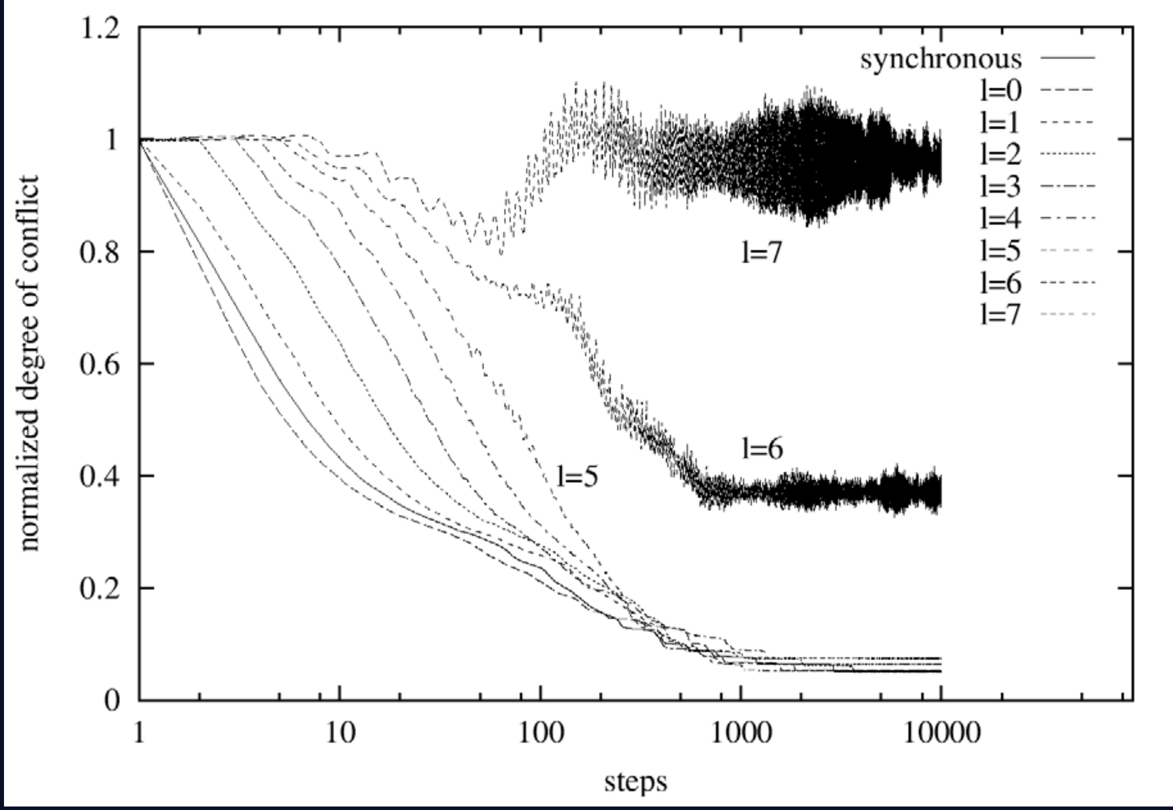

- results shown are for CFP(0.3) on 2D grids with 4 colours subject to various amounts of message randomization
- similar results were obtained for small amounts of message loss

# Asynchronous Execution

- The synchronous FP algorithm requires synchronization, which may:
  - require overhead (e.g. communication cost)
  - slow down the process (wait for the slowest message and node)
  - slow down convergence — or not
- For asynchronous FP the essential idea is the same as for synchronous version, except that execution is asynchronous:
  - *Non-synchronized* infinite loop (but same rate for all nodes):
    - probabilistic activation
      - a node activates if a randomly generated number falls below some fixed activation level p
    - if a node activates, it non-deterministically chooses its next colour
      - it computes a histogram of colour usage among its neighbours, based on what *it last heard from them*
      - it then chooses any colour that is least used in the histogram
      - if the chosen colour differs from its current colour, it tells its neighbours
- Asynchrony may help in symmetry breaking, but communication latency may cause ill-advised changes

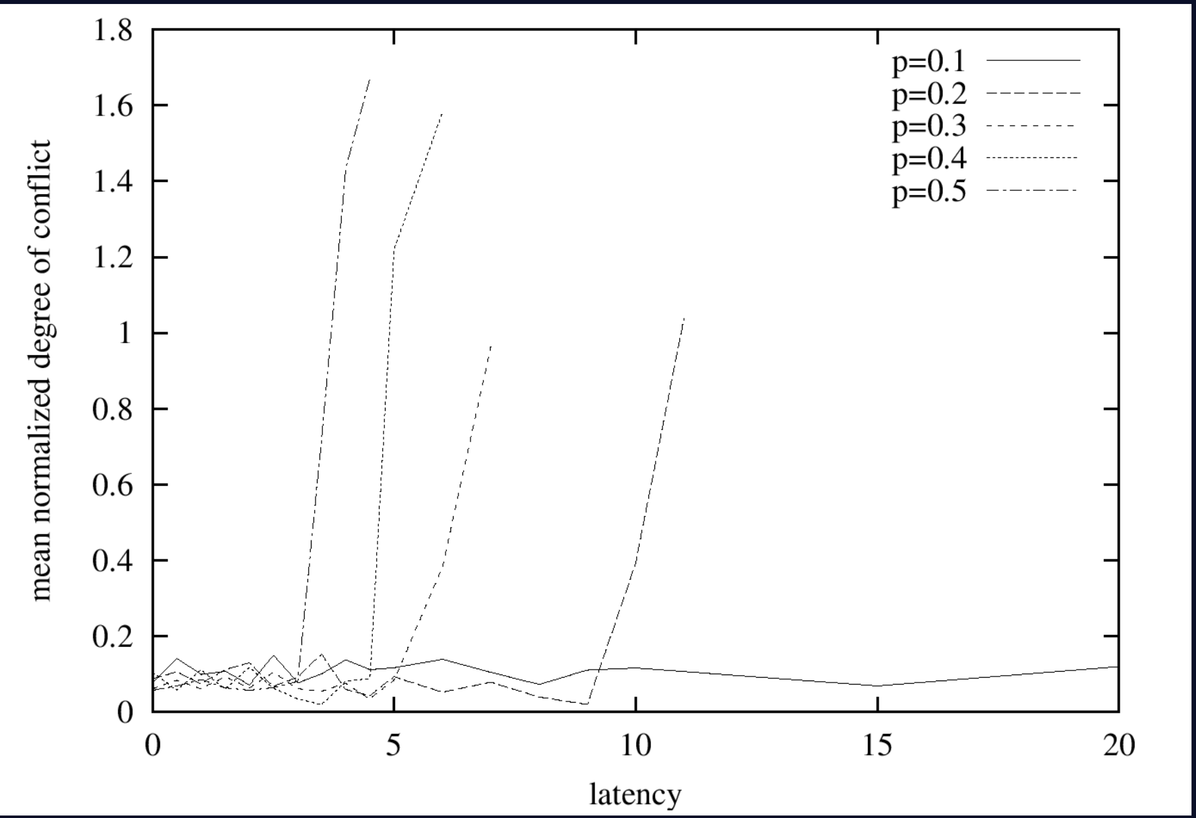
# Effect of Communication Latency

- Performance of asynchronous FP is reasonable for moderate latencies
  - short-term performance degrades (as expected)
  - long-term result quite good
- Performance is even better than synchronous FP when latency < 0.5 time units
- Performance sharply becomes very poor for higher latencies
  - divergence
  - latency = 7 not better than random colouring




- experimental results averaged for 20 random graphs
- p = 0.3
- mean degree = 10
- chromatic number = 3

# Communication Latency and Activation Probability




- Sharp performance drop for higher latencies: the threshold latency decreases as activation probability increases
- This is due to higher probability of “collision” : a colour-change message still travelling along an edge when decision is taken

- degree of conflict averaged over 10,000 steps
- mean degree = 10
- chromatic number = 3

# Effect of Collision Probability

- For activation probability p and latency L,(an upper bound on) the probability of collision is about

  $1 - (1 - p)^L$

- Performance drop indeed depends on collision probability: fine up to about 0.8; bad at 0.9 and higher
- So given latency L, a safe activation probability is:

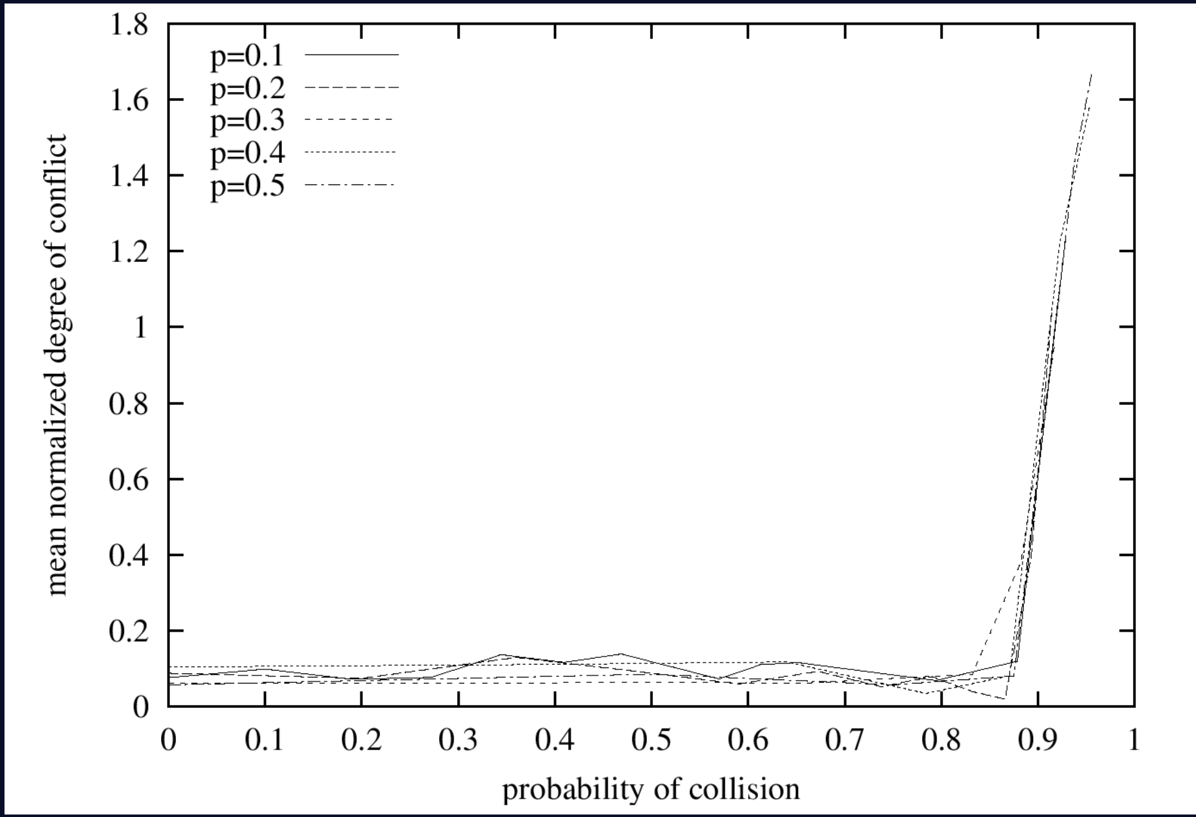
  $p \leq 1 - 0.2^{1/L}$

$L = 1 \rightarrow p \leq 0.80$

$L = 2 \rightarrow p \leq 0.55$

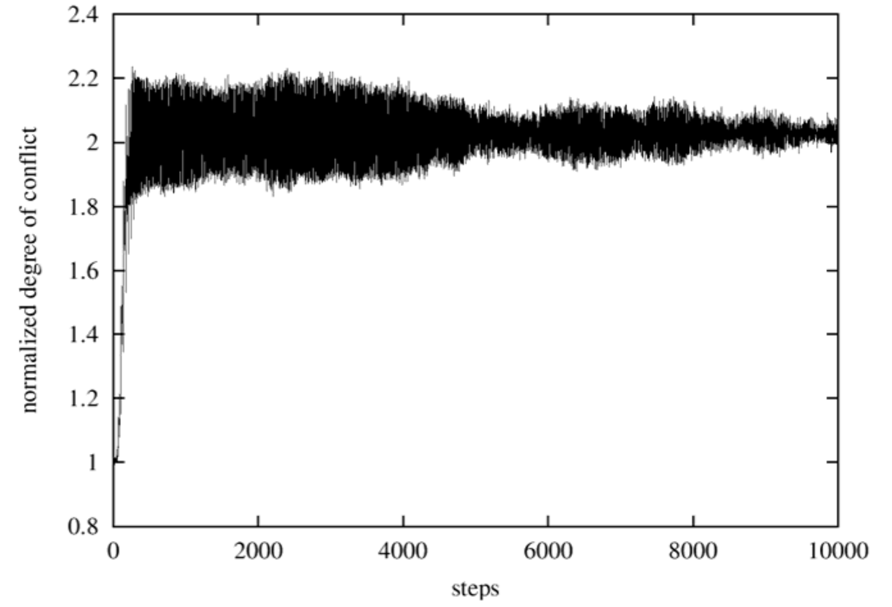
$L = 4 \rightarrow p \leq 0.42$

$L = 8 \rightarrow p \leq 0.18$



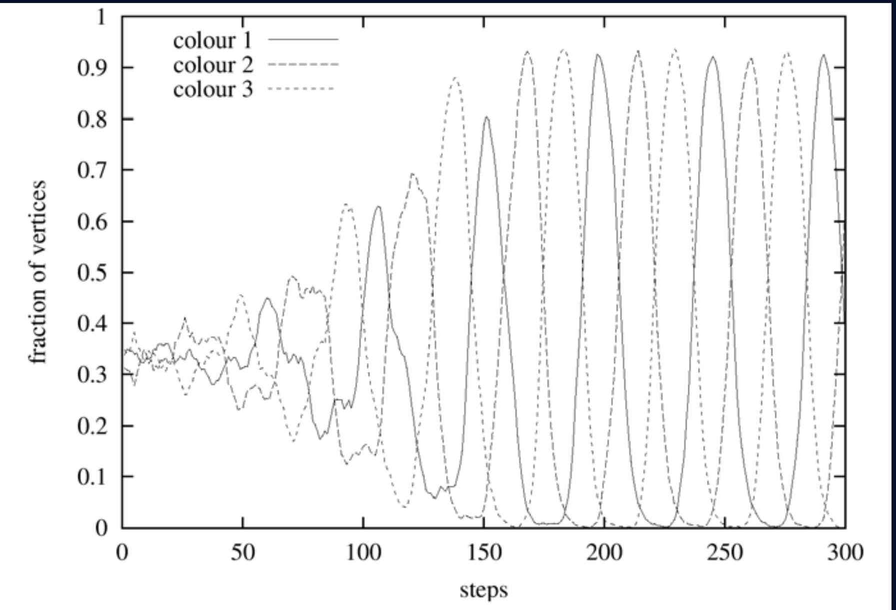

- degree of conflict averaged over 10,000 steps
- mean degree = 10
- chromatic number = 3

# Very High Latencies




- p = 0.3
- L = 15

- Surprise: for very high latencies, the normalized degree of conflict Γ tends to a mean value of approximately 2




- p = 0.3
- L = 10

- For very high latencies, the control mechanism gets caught in an out-of-phase, oscillating trajectory, with period > 2L

# Conclusion

- The FP algorithm is simple but effective for distributed, real-time, approximate colouring of sparse graphs
  - scalable, low-cost, robust
- Basic framework of stochastic activation & local optimization seems appropriate for other distributed constraint problems
  - graph colouring serves as a clean, archetypal problem
- The algorithm has also been tested with dense, random graphs
  - interesting, but different, results
  - proper k-colourings quickly obtained for very dense k-colourable graphs
    - local constraints guide colouring to a unique, proper colouring
- Asynchronous execution and communication latency are handled well
  - provided that the activation probability does not exceed a critical level
- Further work on algorithm
  - non-uniform activation levels, perhaps determined dynamically from local metrics