An Experimental Assessment of a Stochastic, Anytime, Decentralized, Soft Colourer for Sparse Graphs

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Outline:

- Motivation: real-time coordination of sensors in a high-latency network
- Modeling coordination as graph colouring
- Soft graph colouring for real-time responsiveness
- A class of stochastic, distributed, anytime algorithms for soft colouring
- Convergence
- Tightness of constraints: deterministic & conservative variants
- Scalability
- Robustness

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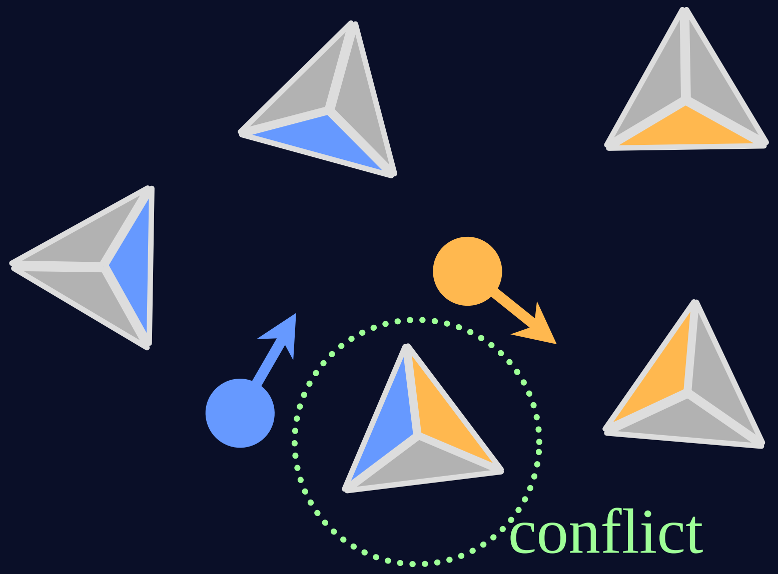
Motivation: Large Networks of Short-Range Sensors

- Short-range, directional radars
  - each can scan 1 of its 3 sectors at a time
  - each scan acquires range & radial velocity
  - battery-operated – conservation important
- Collaboration needed for tracking
  - 3 approximately-simultaneous scans needed for trilateralization
- Low-power radio communication
  - low bandwidth, high latency
  - reveals positions of radars – minimize

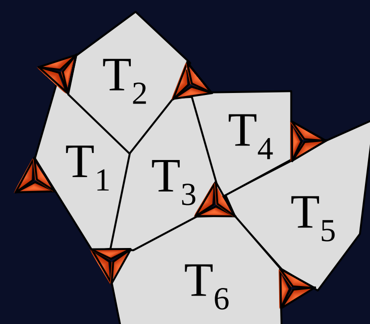


- Coordination mechanism organizes collaboration
  - optimizes simultaneous scanning, minimizes costs
- Must be:
  - scalable (e.g., to 10^5 sensors)
  - real-time adaptive (e.g., new targets are detected, existing targets disappear)
  - robust (e.g., hardware may fail)

Inter-Sensor Collaboration

- Main requirement: scan each target simultaneously with 3 radars
  - define virtual resources: *trackers*
  - each tracker is comprised of 3 sectors on nearby radars
    - $T_i \equiv \{R_{i1}:S_{i1}, R_{i2}:S_{i2}, R_{i3}:S_{i3}\}$
  - each tracker can track a single target over some contiguous region
- Main constraint: each radar can scan only 1 sector at a time
  - if two trackers use different sectors on the same radar, they are mutually exclusive
    - $\text{mutually\_exclusive}(T_1, T_2) \Leftrightarrow \exists\, j,k \in \{1, 2, 3\}: R_{1j}=R_{2k} \wedge S_{1j} \neq S_{2k}$
- Compute a cyclic schedule of tracker usage
  - worst-case assumption: all trackers need to be used
  - mutually exclusive trackers cannot be used in the same time slot
  - number of time slots determined by target speed, scan time & revisit period

timeslot #	scan start time (seconds)	scan end time (seconds)	T1	T2	T3	T4	T5	T6
1	0.0	2.0	X				X	
2	2.0	4.0		X				X
3	4.0	6.0			X			
4	6.0	8.0				X		



Modeling Coordination as Graph Colouring

- Each tracker can be mapped to a *node* in an undirected graph
- Each mutual exclusion constraint then maps to an *edge*
  - nodes that are *adjacent* in the graph are mutually exclusive/cannot be used simultaneously
  - two nodes are said to be neighbors iff they are adjacent
- A *proper k-colouring* of the graph's nodes maps to a feasible schedule
  - time slot $\Leftrightarrow$ integer in Z_k $\Leftrightarrow$ colour

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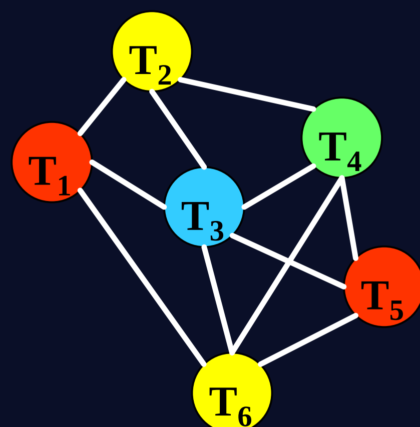
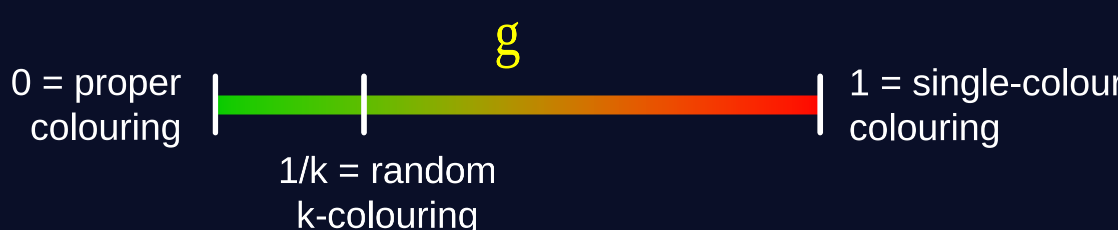


Soft Graph Colouring

- An edge connecting nodes of the same colour represents a *conflict*
  - some radar has been scheduled to scan two sectors simultaneously
- For real-time adaptation, the number of conflicts must be quickly reduced
  - fast reduction to acceptable levels is more important than total elimination
- Define the *degree of conflict* as the fraction of edges that are conflicts
  - let E be the set of edges and C_v the colour of node v

$$g \equiv \frac{\left|\{\{u,v\} \in E \mid C_u = C_v\}\right|}{|E|}$$



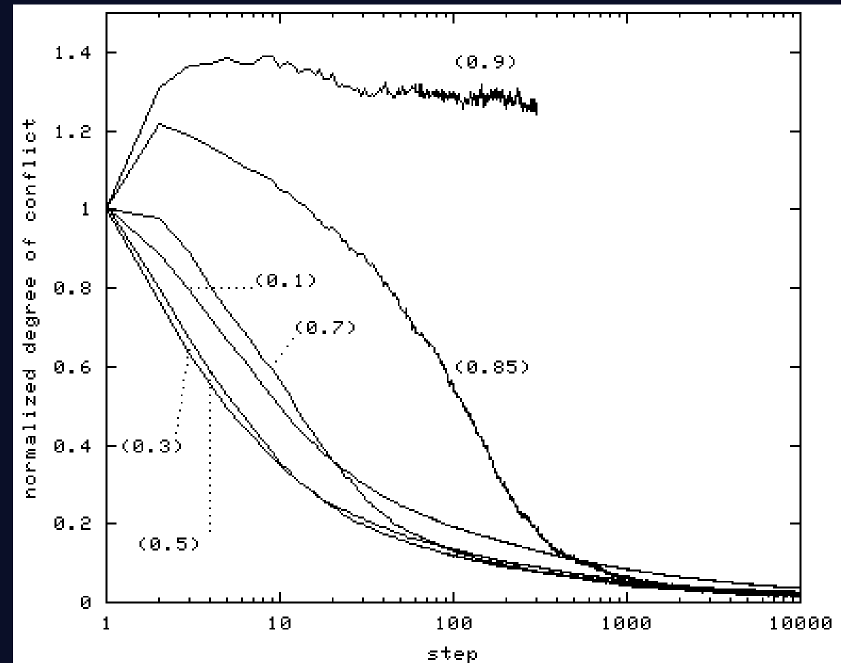
- Normalize: $\Gamma \equiv k\gamma$
  - random k-colouring has an expected Γ of 1
- Assessment of coordination mechanism is based on how quickly it reduces Γ after random initialization

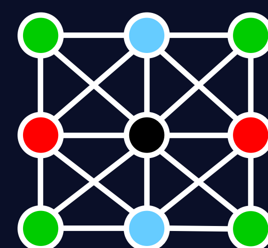
A Class of Distributed, Min-Conflict Algorithms

- Main idea: each node repeatedly chooses its own colour to minimize its conflicts with adjacent nodes
- Fixed Probability algorithm FP(p) …
- Initialization:
  - each node chooses a random colour and informs its neighbours
- Synchronized loop:
  - probabilistic activation
    - a node activates if a randomly generated number falls below some fixed activation level p
  - if a node activates, it non-deterministically chooses its next colour
    - it computes a histogram of colour usage among its neighbours, based on what they last told it
    - it then chooses *any* colour that is least used in the histogram
    - if the chosen colour differs from its current colour, it informs its neighbours
- Convergence?
  - under the right conditions, the total number of conflicts reduces over time and *may* converge to 0 …

Effect of Activation Level on Convergence of FP

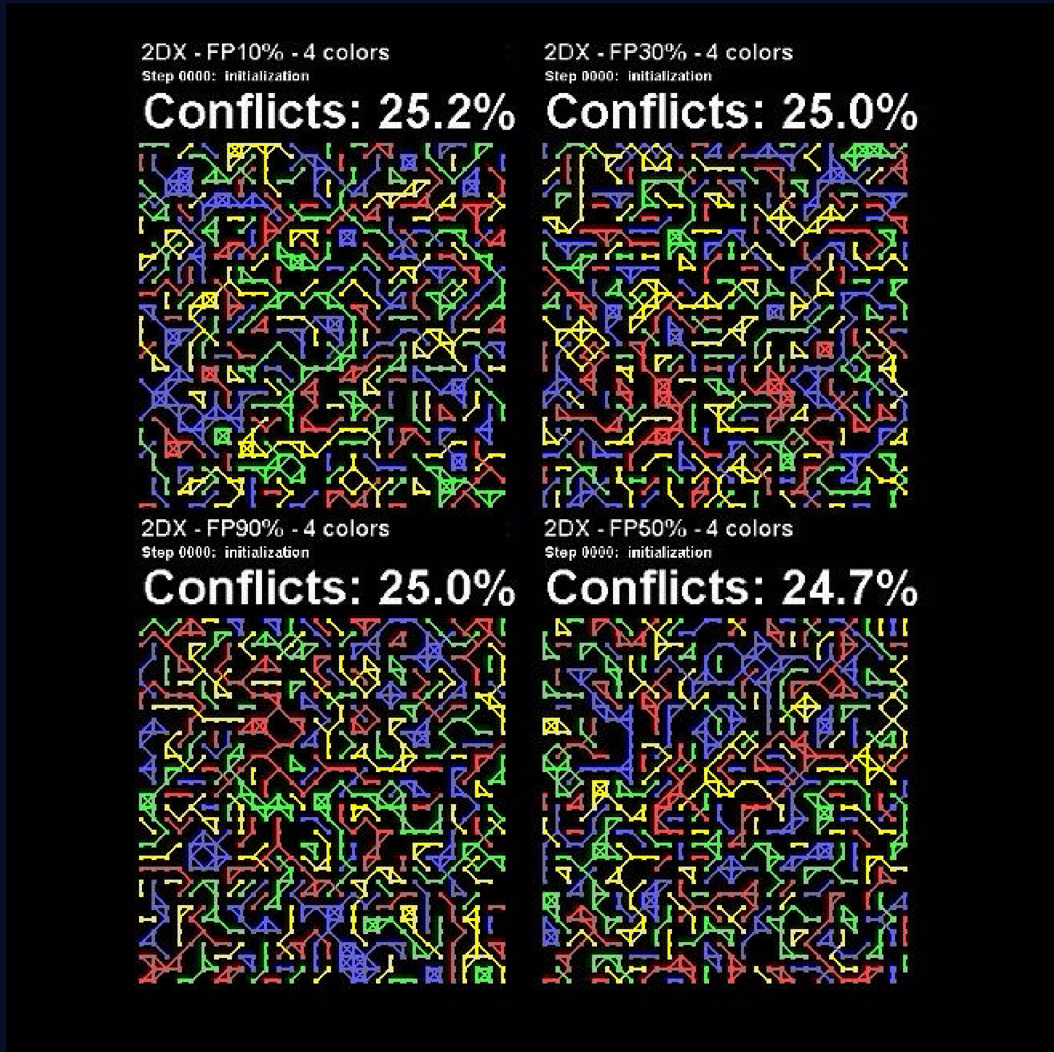
- Measure (normalized) degree of conflict after each synchronous step
  - experiment performed in simulator
- When activation level is too high, thrashing occurs
  - too many neighbours are simultaneously updating colours
  - because of out-of-date information, they make mutually harmful decisions
- When activation level is too low, adaptivity is hindered
  - extreme case is sequential execution
- Need compromise between speed and coherence
  - an activation level of 0.3 seems to be reasonable for sparse graphs
  - this level was used for experiments reported in following slides





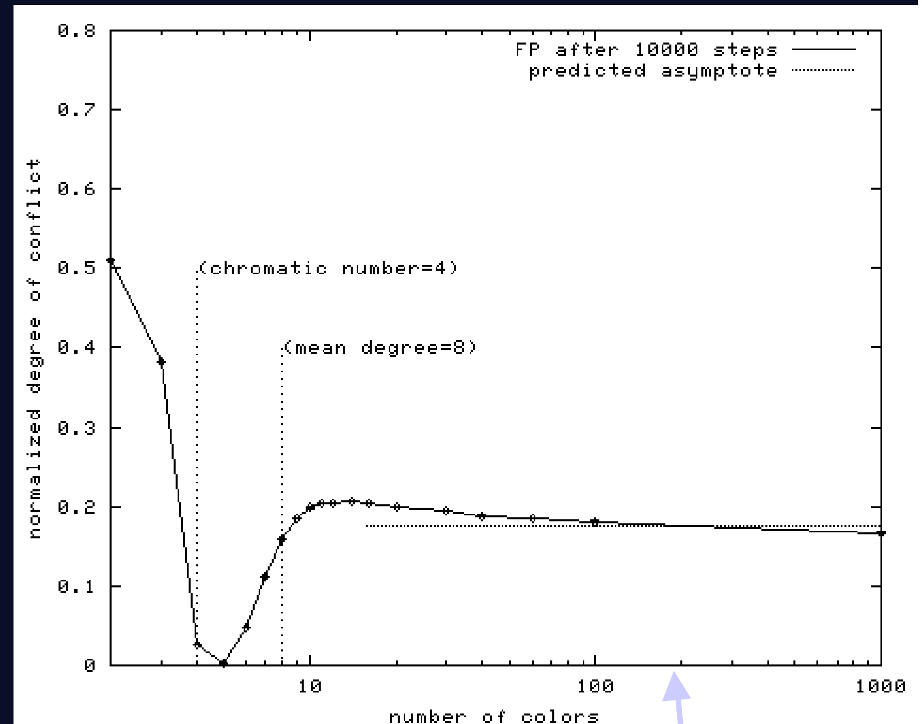
- experimental results shown for 2D grids
  - number of colours = chromatic number = 4
  - 500-5000 nodes
- experiments also performed with random graphs having higher, known chromatic numbers

Animation: Activation Threshold



Effect of Tightness of Constraints

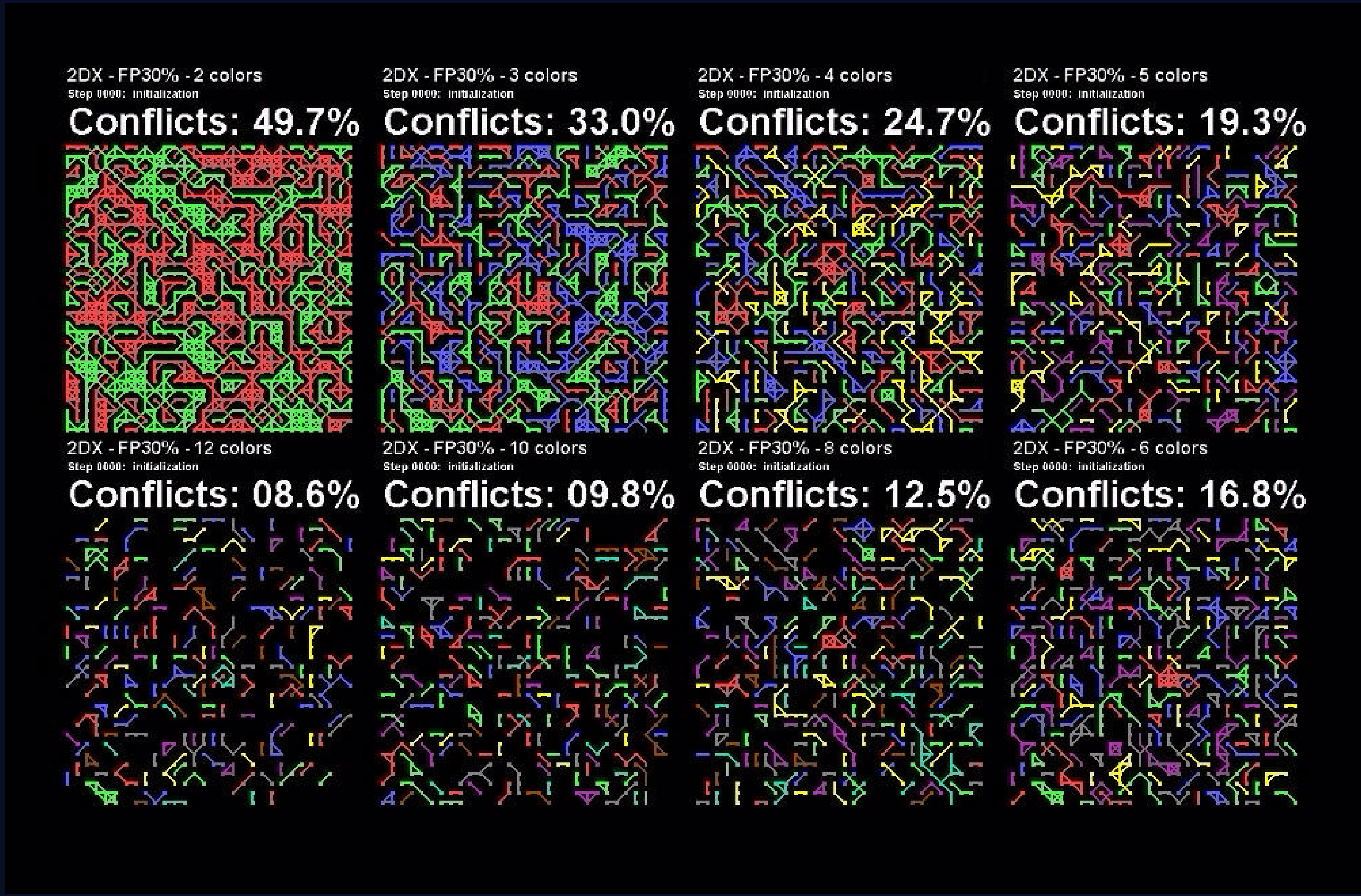
- Performance of FP is good on over-constrained problems
  - where #colours<chromatic number
  - for 2D & 3D grids, observed convergence value of degree of conflict is close to theoretical minimum
- Performance of FP is poor on loosely constrained problems
  - where #colours>>chromatic number
  - intuitively, these are easy problems
- When loosely constrained, each colour choice is essentially random
  - for each given node, most colours are not used by any neighbour
  - FP chooses randomly from among the unused colours
  - asymptotic value predicted as $\alpha/(2-\alpha)$ where α is the activation level



this is *not* a time axis

- experimental results shown for 2D grids
- chromatic number = 4

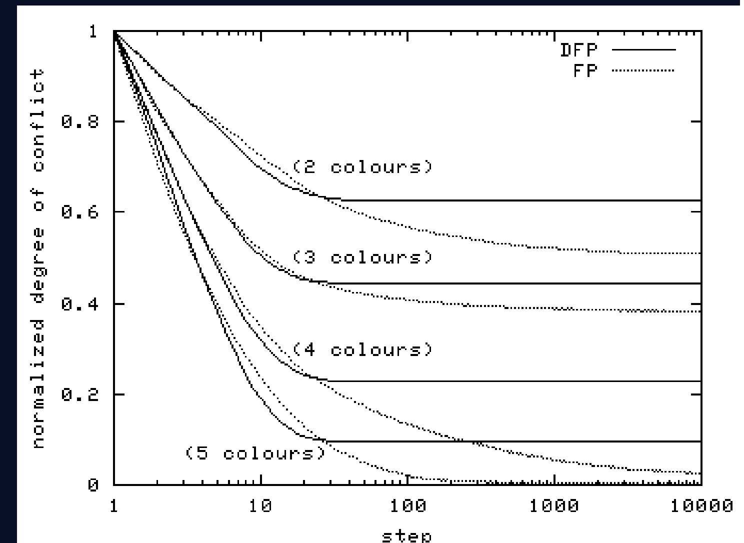
Animation: Tightness of Constraints



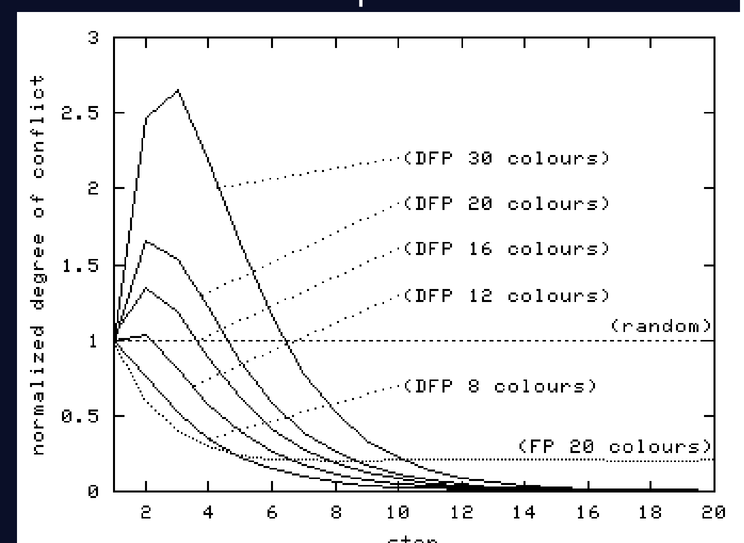
Deterministic Variant

- Possible solution: deterministic colour choice
  - each node chooses the *lowest* colour (in Z_k) that minimizes its conflicts
- Long-term performance:
  - better than FP when loosely-constrained
  - worse than FP otherwise
    - converges to local minimum
    - randomization techniques can improve convergence values, but at the cost of poor short-term performance
- Short-term performance is poor
  - extreme spike in degree of conflict when loosely constrained
    - random initialization causes many neighbours to have the same, unused colour
    - in the next step, those that activate all change to that colour, causing numerous conflicts
  - non-uniform, deterministic choice reduces but does not eliminate this problem

long-term performance



short-term performance

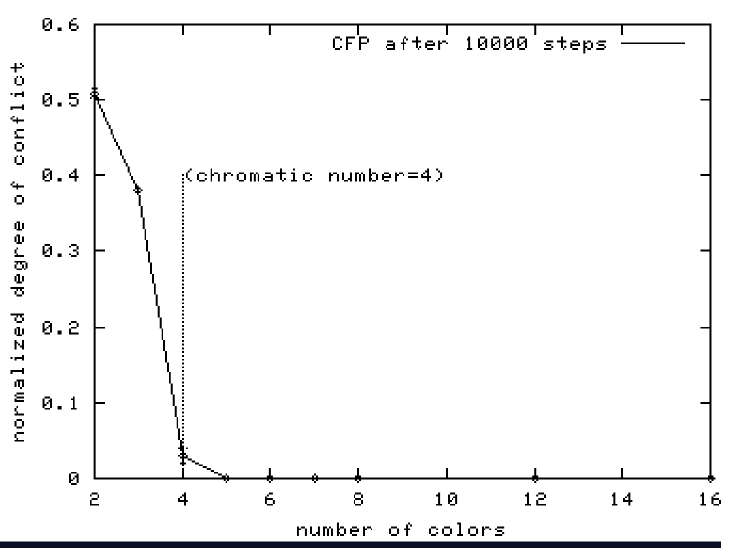


- chromatic number = 4

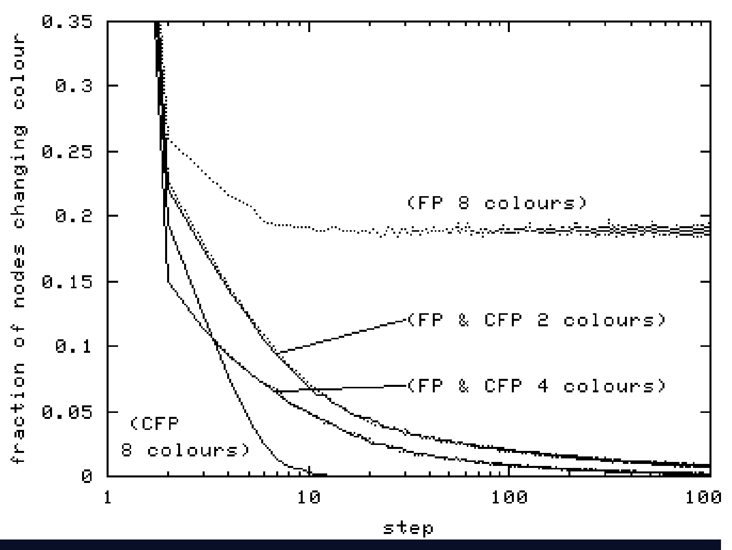
Conservative Variant

- Colour choice is non-deterministic
- But activation is restricted
  - in addition to passing the test for random number<activation level
  - a node may activate only if it has a conflict with any neighbour
- Conservative variant has good performance overall
  - communication costs are also better than FP's for loosely constrained problems
    - under FP, node activity continues unabated forever
    - under CFP, node activity decreases with the degree of conflict

conflicts

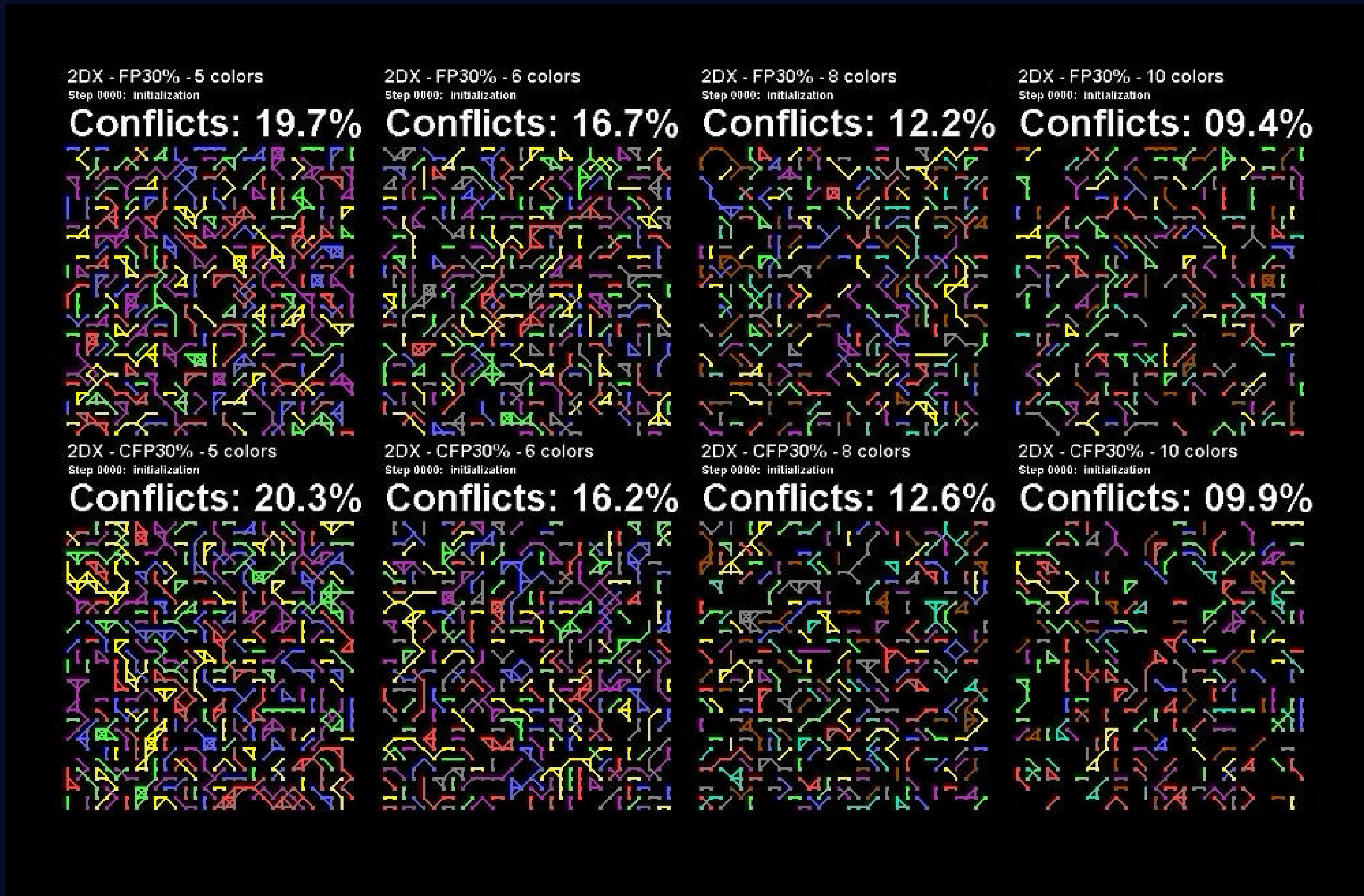


communication rate



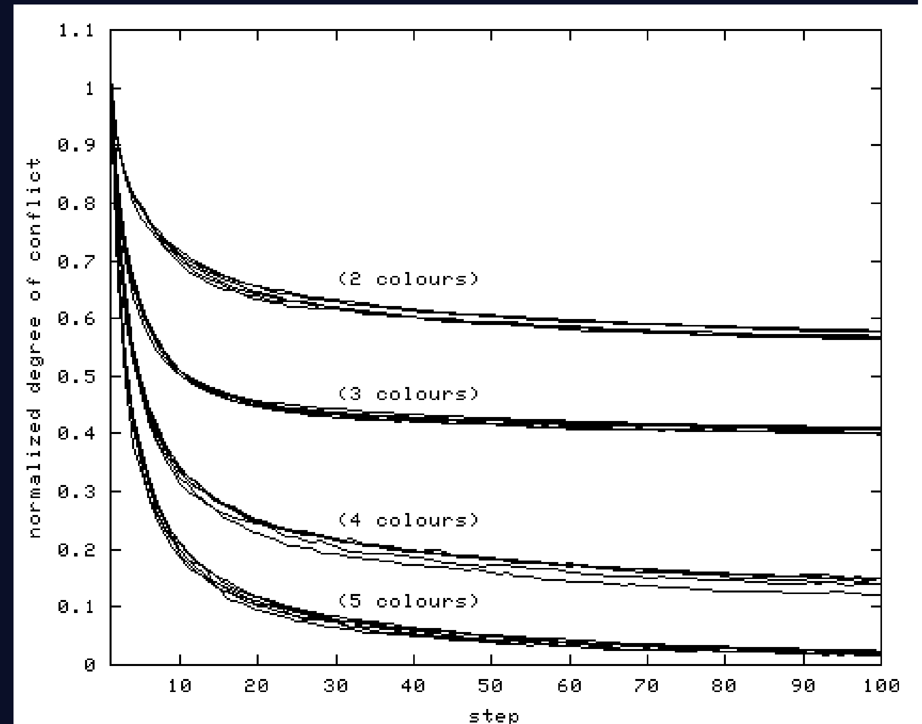
- experimental results shown for 2D grids
- chromatic number = 4

Animation: FP vs. CFP



Scalability

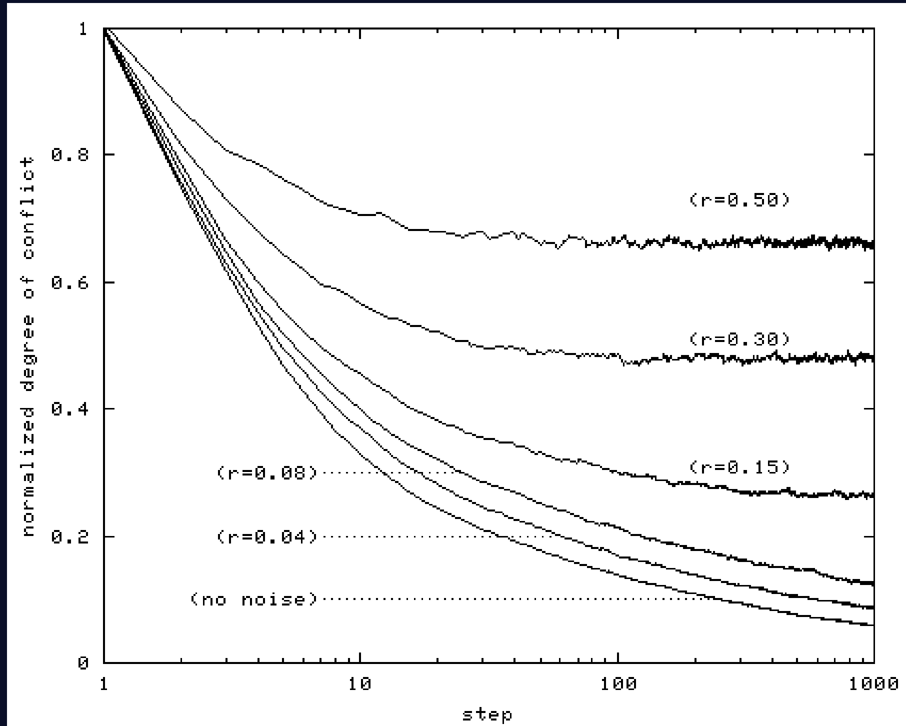
- The algorithm is scalable in cost
  - per node, per step costs depend on (mean) degree of the graph
  - they do not depend on the number of nodes
    - to the extent that the mean degree is independent of the number of nodes
- The algorithm is scalable in performance
  - for large graphs, the reduction in normalized degree of conflict over steps shows little variation for graphs of different sizes



- results shown are for CFP(0.3)
- 6 graphs of different sizes (500-5000 nodes)
  - each graph has chromatic number 4
  - each was coloured using 2, 3, 4 & 5 colours

Robust against Communication Noise

- Each colour-change message subjected to random process:
  - probability r, colour randomized
  - probability d, message lost
  - otherwise, message unchanged
- For small amounts of noise, incremental increases in degree of conflict are observed
  - no catastrophic failure

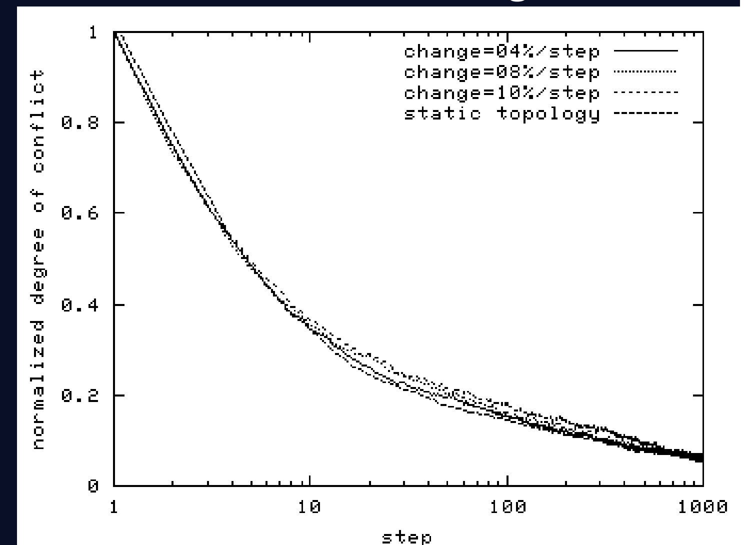


- results shown are for CFP(0.3) on 2D grids with 4 colours subject to various amounts of message randomization
- similar results were obtained for small amounts of message loss

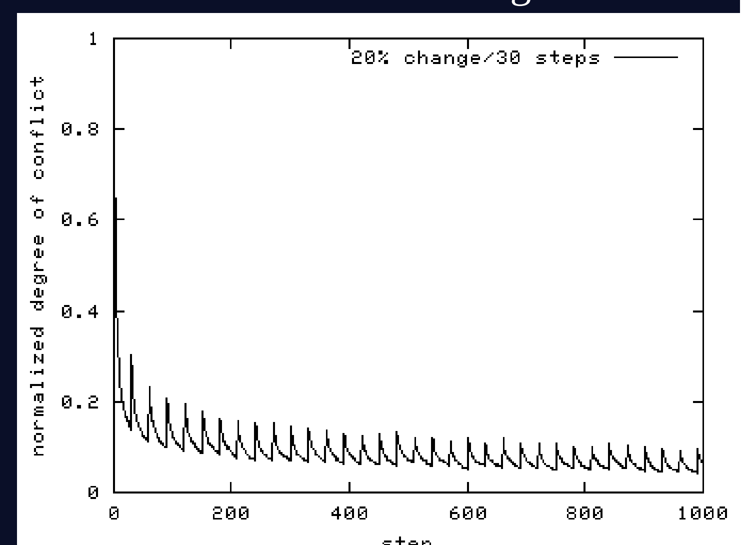
Robust against Topology Change

- Simulate the effects of dynamic hardware availability by varying the topology
  - initially, R nodes (and their incident edges) are removed at random and recorded
  - then, every P steps:
    - another R nodes (and their incident edges) are removed and recorded
    - of the pool of 2R removed nodes, R are selected at random and restored
    - any removed edges whose end nodes are now present in the graph are restored
  - not a complete simulation
    - it does not address the need to reassign tasks that were supposed to be handled by hardware that failed
- Continuous change: P=1, small R
  - little effect
- Intermittent change: P=30, large R
  - spikes in the number of conflicts

Continuous Change



Intermittent Change



Conclusion

- The CFP algorithm is simple but appears to be effective for distributed, real-time, approximate colouring of sparse graphs
  - scalable, low-cost, robust
  - effective on under-, critically- and over-constrained problems
- Basic framework of stochastic activation & local optimization seems appropriate for other distributed constraint problems
  - graph colouring serves as a clean, archetypal problem
- The algorithm has also been tested with dense, random graphs
  - interesting, but different, results
  - proper k-colourings quickly obtained for very dense k-colourable graphs
    - local constraints sufficient to guide colouring to a unique, proper colouring
- Further work on experiments
  - other types of graphs and/or constraints
  - lower bounds for over-constrained problems on random graphs
- Further work on algorithm
  - non-uniform activation levels, perhaps determined dynamically from local metrics